Problem 2.20

[Computer] Use suitable graph-plotting software to plot graphs of the trajectory (2.36) of a projectile thrown at 45° above the horizontal and subject to linear air resistance for four different values of the drag coefficient, ranging from a significant amount of drag down to no drag at all. Put all four trajectories on the same plot. [*Hint:* In the absence of any given numbers, you may as well choose convenient values. For example, why not take $v_{xo} = v_{yo} = 1$ and $g = 1$. (This amounts to choosing your units of length and time so that these parameters have the value 1.) With these choices, the strength of the drag is given by the one parameter $v_{\text{ter}} = \tau$, and you might choose to plot the trajectories for $v_{\text{ter}} = 0.3$, 1, 3, and ∞ (that is, no drag at all), and for times from $t = 0$ to 3. For the case that $v_{\text{ter}} = \infty$, you'll probably want to write out the trajectory separately.]

Solution

The position of a particle in a medium with linear drag is given by the parametric equations in Equation (2.36) on page 54.

$$\begin{cases} x(t) = v_{xo}\tau(1 - e^{-t/\tau}) \\ \\ y(t) = (v_{yo} + v_{\text{ter}})\tau(1 - e^{-t/\tau}) - v_{\text{ter}}t \end{cases} \tag{2.36}$$

Solving the first equation for t and plugging it into the second equation gives Equation (2.37) on page 54 (see Problem 2.17), which is more convenient to plot.

$$y(x) = \frac{v_{yo} + v_{\text{ter}}}{v_{xo}}x + v_{\text{ter}}\tau \ln\left(1 - \frac{x}{v_{xo}\tau}\right) \tag{2.37}$$

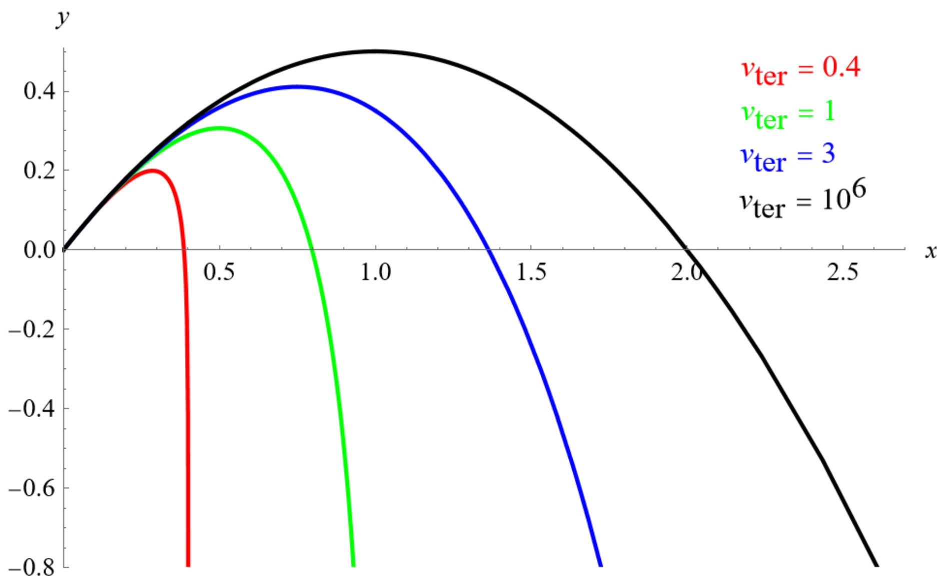
Eliminate $\tau = m/b$ in favor of $v_{\text{ter}} = mg/b$.

$$\begin{aligned} y(x) &= \frac{v_{yo} + v_{\text{ter}}}{v_{xo}}x + v_{\text{ter}}\frac{g\tau}{g} \ln\left(1 - \frac{gx}{v_{xo}g\tau}\right) \\ &= \frac{v_{yo} + v_{\text{ter}}}{v_{xo}}x + v_{\text{ter}}\frac{v_{\text{ter}}}{g} \ln\left(1 - \frac{gx}{v_{xo}v_{\text{ter}}}\right) \\ &= \frac{v_{yo} + v_{\text{ter}}}{v_{xo}}x + \frac{v_{\text{ter}}^2}{g} \ln\left(1 - \frac{gx}{v_{xo}v_{\text{ter}}}\right) \end{aligned}$$

In order for the projectile to have a 45° angle above the horizontal, it's necessary that $v_{xo} = v_{yo}$. For convenience, set $v_{xo} = v_{yo} = g = 1$.

$$y(x) = (1 + v_{\text{ter}})x + v_{\text{ter}}^2 \ln\left(1 - \frac{x}{v_{\text{ter}}}\right)$$

This equation is plotted versus x for four values of v_{ter}.



The trajectory is a parabola when there's no drag. The more significant drag is, the more quickly the projectile loses its horizontal velocity.